FACIAL BEAUTY AND FRACTAL GEOMETRY

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Abstract

What is it that makes a face beautiful? Average faces obtained by photographic[1−4] or digital[5] blending are judged attractive[1−5] but not optimally attractive[6,7] — digital exaggerations of deviations from average face blends can lead to higher attractiveness ratings[7]. My novel approach to face design, however, does not involve blending at all. Instead, the image of a female face with high ratings is composed from a fractal geometry based on rotated squares and powers of 2. The corresponding geometric rules are more specific than those previously used by artists such as Leonardo and Dürer. They yield a short algorithmic description of all facial characteristics, many of which are compactly encodable with the help of simple feature detectors similar to those observed in mammalian brains. This suggests that a face's beauty correlates with simplicity relative to the subjective observer's way of encoding it.

Each of 14 test subjects (10 male, age 26-37 yr; 4 female, age 16-46 yr) rated the artificial female face depicted in Figure 1 as "beautiful" and as more attractive than any of the six faces in a previous study based on digital face blends[7]. The design principle is clarified in Figure 2. First the sides of a square were partitioned into 2^4 equal intervals; then certain obvious interval boundaries were connected to obtain three rotated, superimposed grids (thick lines in Figure 2) with rotation angles $\pm arcsin\ 2^{-3}$ and $arcsin\ 2^0 = 45°$. Higher-resolution details of the fractal[8] grids were obtained by iteratively selecting two previously generated, neighbouring, parallel lines and inserting a new one equidistant to both (for clarity, some fine-grained detail is omitted in Figure 2). Shifted copies of circles (also omitted for clarity) inscribable in thick-lined squares of Figure 2, scaled by powers of 2, account for transitions between non-parallel face contours such as facial sides and chin. To achieve a realistic impression, some colors/greyscales were first matched to those in the photograph of a real person, then further edited digitally. Figures 3–5 isolate important feature-defining lines of the first, second, and third self-similar[8] grid, respectively. Hundreds of alternative simple geometric designs I tried led to less satisfactory results.

The face satisfies several ancient rules used by mathematically oriented artists such as Leonardo da Vinci and Albrecht Dürer, e.g.: it is symmetrical; the distance between the eyes equals one eye-width or one nose-width; the tip of the nose is about half the way from chin to eyebrows. Certain facial measurements that correlate with attractiveness[9,10] are automatic by-products. The present feature-defining rules, however, are much more specific than those used in previous work. For instance, they specify (compare Figures 2 and 3): (1) The prolongations of the left eyebrow's lower and upper edges meet the right eye lid and the right eyeshadow's upper edge, respectively. (2) The upper edge of the left eye's nose-near interior and the lower edge of the right eye's lie on the same line through the image center between the pupils. (3) Squares of equal size are formed by lines defining (3a) upper edges of left eyebrow and left nostril, left side of nasal ridge, and left forehead; (3b) upper edge of left eyebrow, lower edge of right nostril, left part of nasal ridge, and

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[*] In particular, images herein may not be used for any commercial purposes (e.g., as templates for computer animation or plastic surgery) without explicit written consent by the author.

right facial boundary; (3c) upper edges of right upper lip and the left eye's nose-near interior, left nasal ridge, and right facial boundary. Rules such as (1–3) above symmetrically hold for opposite face parts (Figure 4). Figure 5 shows how large diagonal squares (with low fractal resolution) shape additional important features, such as left and right parts of lower lip and chin, and certain contours of eyes and eyebrows. Figure 2 also specifies many additional simple facial proportions based on powers of 2.

The attractiveness of the face can be explained in the spirit of a previous suggestion[11] based on the theory of minimum description length[12–16]: *among several patterns classified as "comparable" by some subjective observer, the subjectively most beautiful is the one with the simplest (shortest) description, given the observer's particular method for encoding and memorizing it.* For instance, realistic representations of female faces must satisfy certain constraints concerning shape and size of defining features. Among images subjectively classified as being within the acceptable tolerances the ones with compact codes will be preferred.

Tentatively accepting this hypothesis, since different humans with face-memorizing strategies tuned by different subjective experiences tend to have similar preferences, we are led to ask: what is common among human face-encoding algorithms? Nobody really knows, but at least it is well-known that early visual processing stages of mammalian brains employ rotation-sensitive edge and bar detectors. According to standard information theory[17] such detectors could contribute to an efficient code of the particular face in Figure 1, because they allow for compressed descriptions of certain important features. For example, detectors that will be maximally responsive to the near-vertical eyebrow contours and parallel eye and mouth contours will respond minimally to the orthogonal contours of nose and facial sides — with high probability they will be either on or off. Loosely speaking, this corresponds to one bit of information as opposed to several bits necessary for describing intermediate values, and is reminiscent of results obtained by a recent learning algorithm for artificial neural nets that also favors codes based on features with sufficient but minimal information content[18,19].

Similarly, hypothetical detectors measuring ratios of distances between parallel lines could help to further compress the description of Figure 1, because the same distance ratios (based on powers of 2) recur over and over. More generally, different parts of the face are encodable by reusing descriptions of other parts, thus allowing for compact algorithms describing the whole: the defining features of the face have low algorithmic complexity[12–14], given a description model[15,16] based on simple geometric feature detectors. Hence the image may be viewed as an example of low-complexity art[11].

Mathematicians find beauty in a simple proof with a short description in the formal language they are using. Facial attractiveness may reflect a similar correlation between beauty and subjective simplicity. Our results indicate that neither average faces obtained by blending nor certain attractive, digital caricatures thereof[7] are as attractive as a particular face whose essential features are compactly encodable using a simple but novel geometric construction method. Future analysis of attractive face types other than the one in Figure 1 may profit from examining the significance of other, especially 3-dimensional, fractal geometric patterns. Generation of aesthetic faces by artists may also provide clues as to how human face recognition works.

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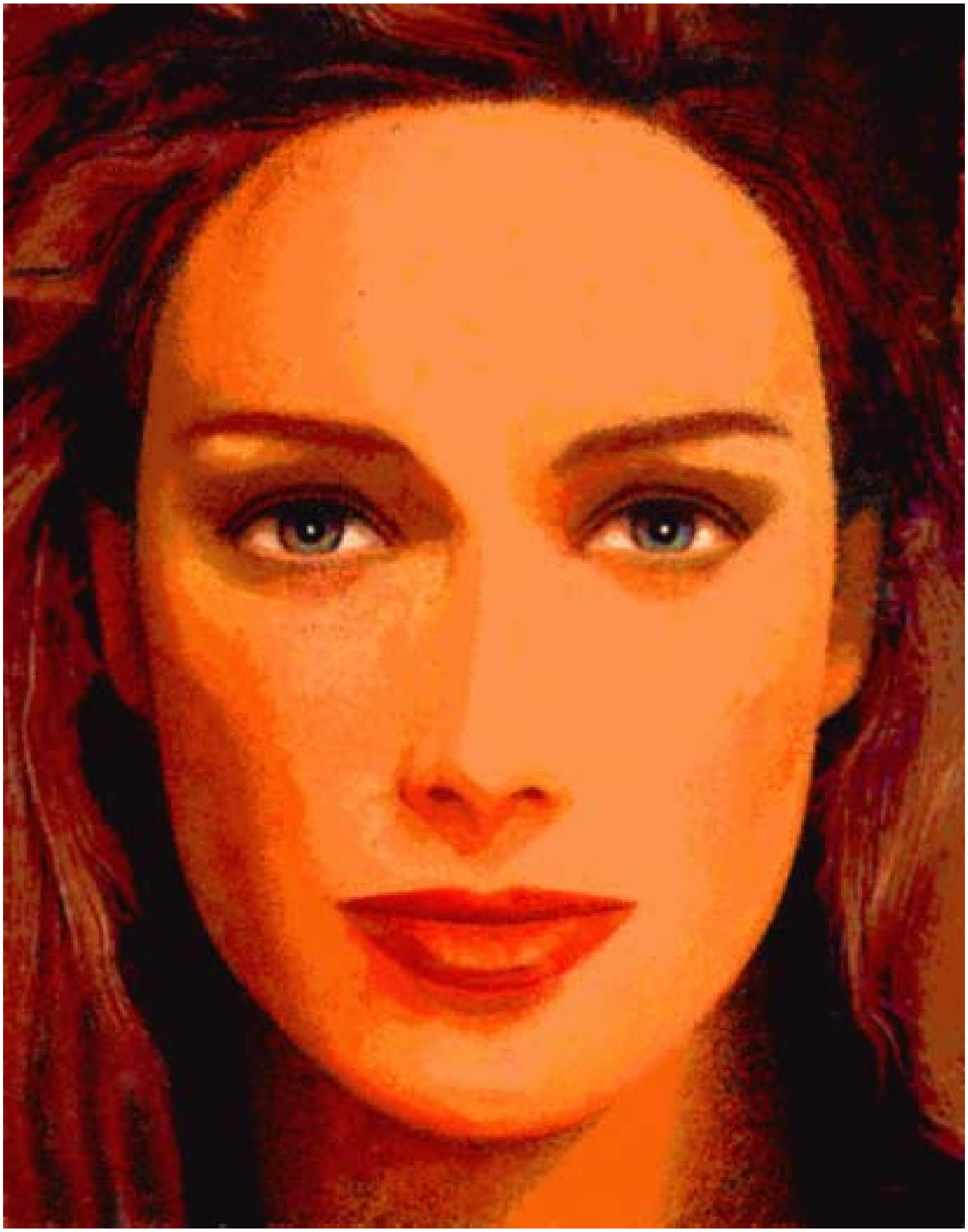

Figure 1: *Color image (digital postscript version) of artificial female face designed according to rules illustrated in Figures 2–5.*

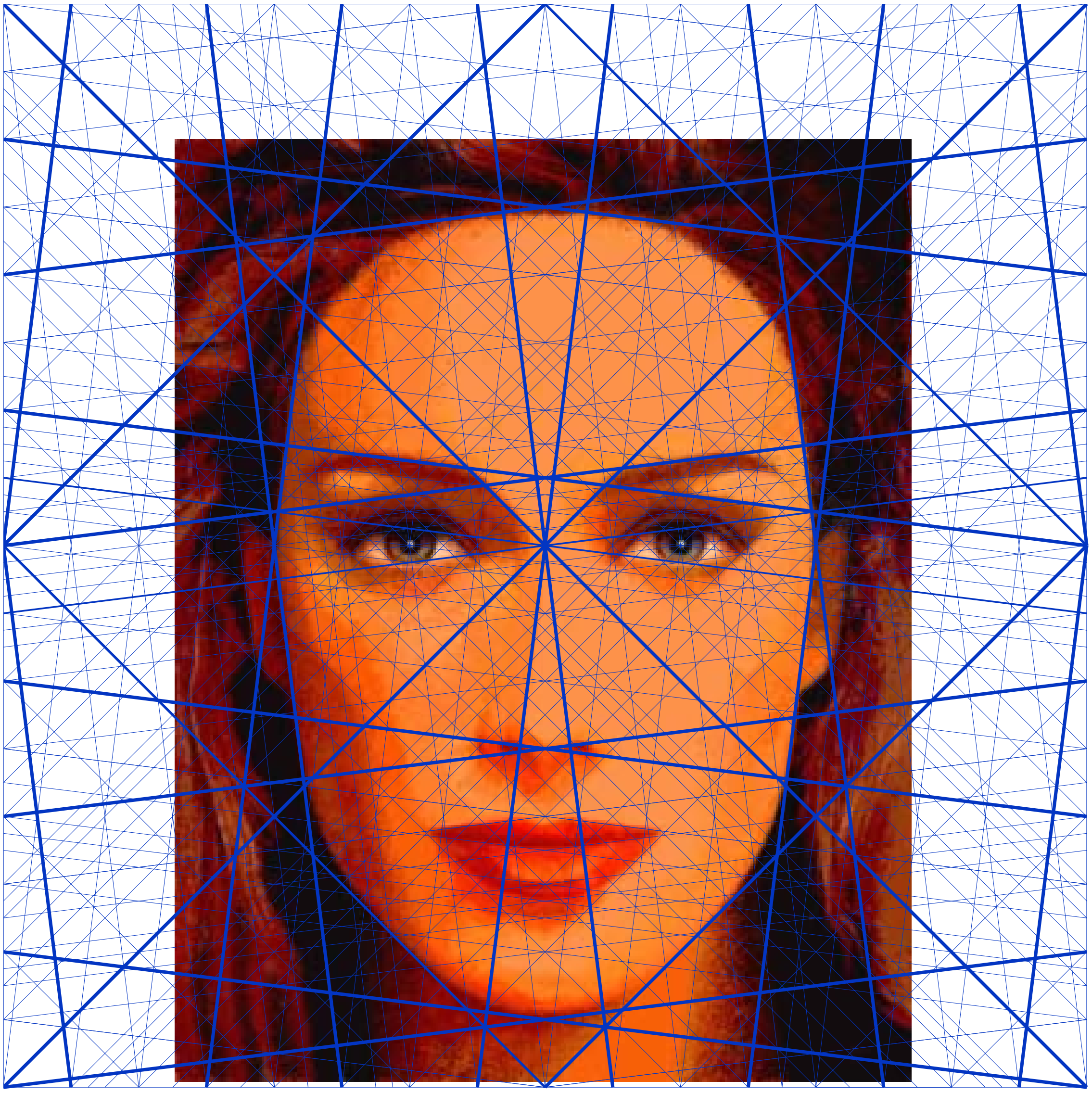

Figure 2: *Explanation of the short algorithm describing the essential features of Figure 1's low-complexity face based on 3 superimposed grids. The scheme enforces simple facial proportions based on powers of 2. The sides of the quadratic frame are partitioned into* 2^4 *equal intervals. Large squares (thick lines) are responsible for basic contours, smaller ones (by factors of* 2^{-n}, $n = 1, 2, \ldots$*) for details. Shifted copies of circles (omitted for clarity) inscribable in thick-lined squares of Figure 2, scaled by powers of 2, account for transitions between non-parallel face contours such as facial sides and chin.*

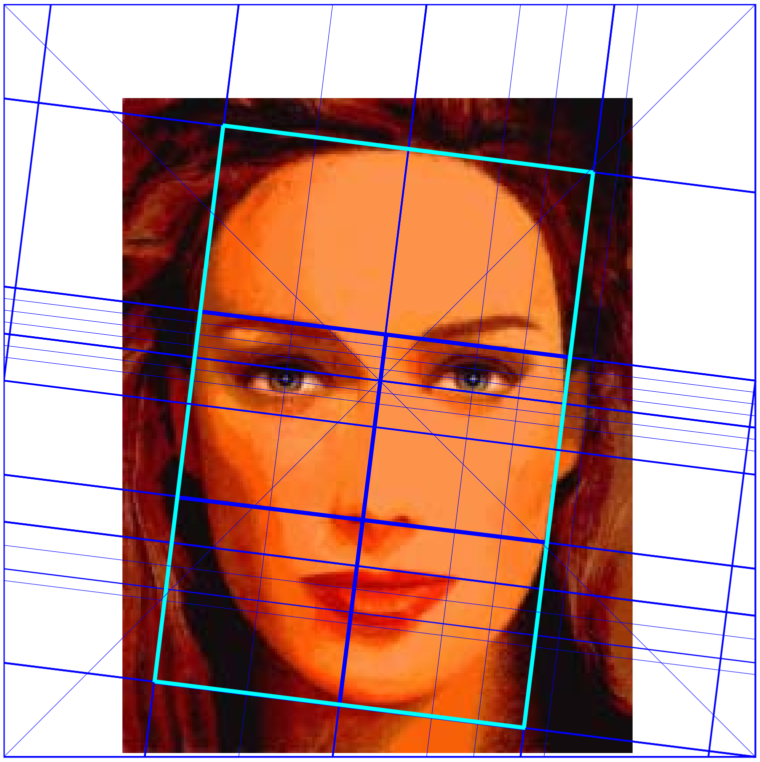

Figure 3: *Selected feature-defining lines of the grid rotated by* $arcsin\frac{1}{8}$.

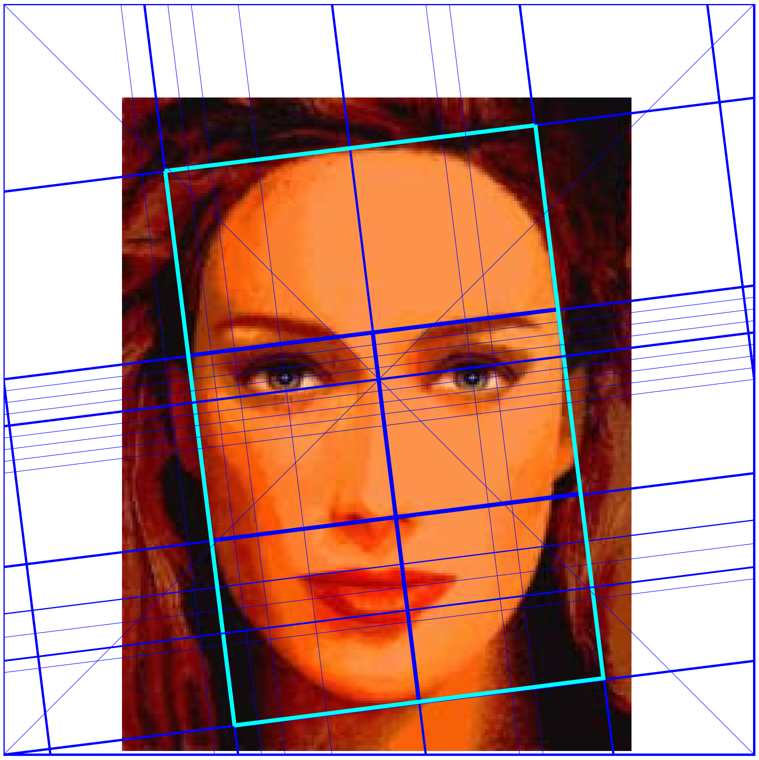

Figure 4: *Selected lines of the grid rotated by* $-arcsin\frac{1}{8}$.

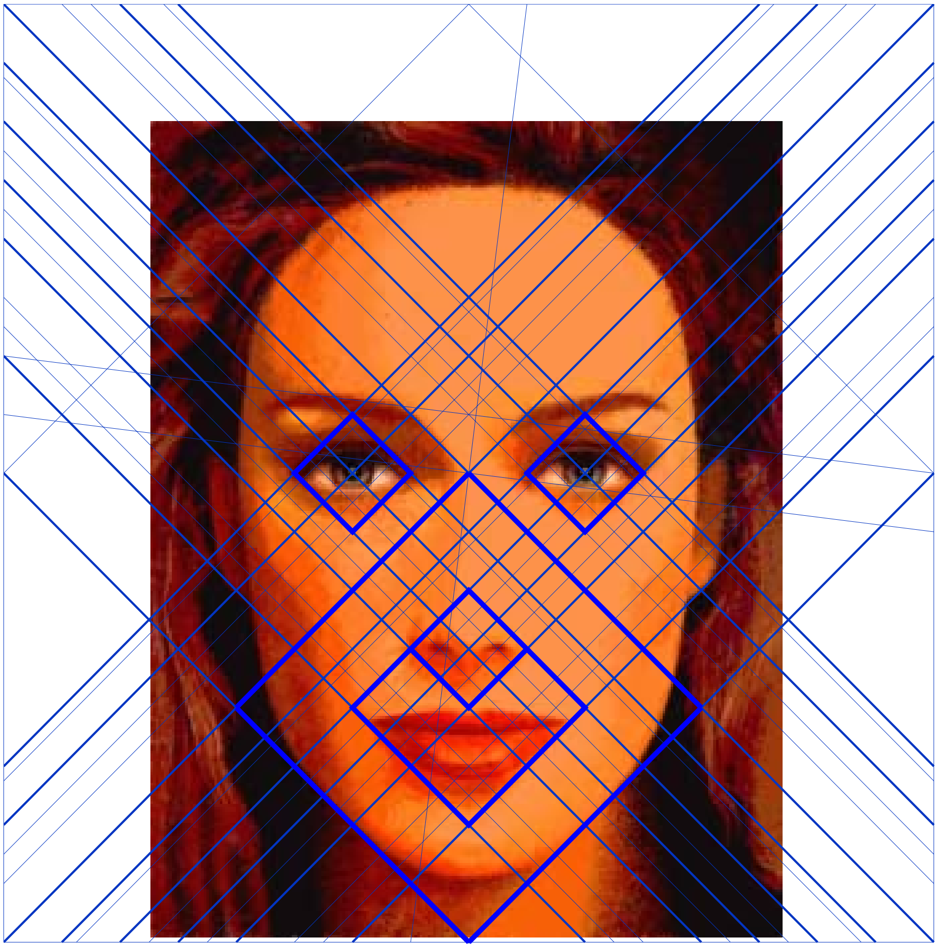

Figure 5: *Selected lines of the grid rotated by* 45°.