

## Eye Spacing Measurement for Facial Recognition

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### Abstract

Few approaches to automated facial recognition have employed geometric measurement of characteristic features of a human face. Eye spacing measurement has been identified as an important step in achieving this goal. Measurement of spacing has been made by application of the Hough transform technique to detect the instance of a circular shape and of an ellipsoidal shape which approximate the perimeter of the iris and both the perimeter of the sclera and the shape of the region below the eyebrows respectively. Both gradient magnitude and gradient direction were used to handle the noise contaminating the feature space. Results of this application indicate that measurement of the spacing by detection of the iris is the most accurate of these three methods with measurement by detection of the position of the eyebrows the least accurate. However, measurement by detection of the eyebrows' position is the least constrained method. Application of these techniques has led to measurement of a characteristic feature of the human face with sufficient accuracy to merit later inclusion in a full package for automated facial recognition.

### Introduction

#### Automated facial recognition

Facial recognition has intrigued researchers for many years. Much study has been made of human perception and a number of automated techniques have been proposed to accomplish this task. An early attempt<sup>1</sup> concerned a device which learnt the pattern of each face using a network of photomultipliers. The device could discriminate between ten photographs of faces following 250 presentations of each photograph. The device appears to have learnt to recognise the pattern of each face and hence was presumably not restricted to facial recognition alone. Later attempts concerned automatic recognition using the facial profile. One method<sup>2</sup> concerned description of the facial profile using 17 fiducials whose components (nose protrusion, chin length etc) were formed from the average of three sittings. Recognition accuracy for a fourth presentation was 96% with the correct result second in an ordered list of the remaining 4% failures. Another method<sup>3</sup> concerned recognition via a profile silhouette whose twelve components were obtained using a circular autocorrelation function. Recognition accuracy is quoted as 90% in a ten class problem which is comparable to the measured performance of human observers. A recent approach has employed artificial neural nets<sup>4</sup> and following a learning process has been demonstrated to distinguish between different subjects.

Geometric measurement of features within a face has been identified as a major part of the human facial recognition process<sup>5,6</sup>. Indeed a set of criteria was developed<sup>6</sup> and used as a basis for recognition by a panel of observers who classified faces according to these criteria using three portraits of each face comprising a frontal, side and a three-quarter profile. Following classification a number of criteria were rejected as being unsuitable for recognition. The measures for each face of the remaining criteria were then sorted by computer which usually required six sortings to isolate a single face.

The set of features includes most basic features of a human face and comprises 21 features including length and protrusion of ears, lip thickness, eye separation and shade, nose profile and mouth width. Eye spacing measurement is attractive for purposes of automated recognition because the eyes offer a distinct feature within a face and which, though sometimes partly obscured by glasses, generally present a common shape (especially since their shape is little altered by changes in facial expression). Recent methods<sup>7</sup> have used natural geometric properties of the eye as a basis for recognition. This paper presents the measurement of the spacing between the eyes by use of the Hough transform technique to locate analytically defined shapes which approximate distinct parts of the eye.

#### Application of the Hough transform

#### The Hough transform

Many techniques have been developed for feature extraction in images though few are as

tolerant to noise as the Hough transform technique<sup>6</sup>. Early versions of this technique located analytically described shapes using the magnitude of gradient information alone. Later developments include use of directional information as provided by a gradient operator to aid the recognition process and for some applications such inclusion provides basis for detection of arbitrarily specific shapes which have no analytic definition.

Application of the Hough transform requires firstly feature extraction to be applied to an image to generate a feature space in which the analytically defined shape is to be located. An array of accumulators is defined in which each element is indexed by a particular set of values for each parameter. The dimensions of this array are defined by the expected bounds on each parameter. Each point in the array of accumulators is incremented for each point in the feature space which is on the analytically defined shape described by that set of parameters and which is satisfactory both in terms of gradient strength and direction.

Consider a point in an image with coordinates  $x'$ ,  $y'$  which is on a shape described by a set of parameters, parameter 1, parameter 2 etc., according to an analytic definition, then

$$\text{feature}(\text{parameter } 1, \text{parameter } 2, \dots) = \text{feature}(\text{parameter } 1, \text{parameter } 2, \dots) + 1$$

iffi the gradient magnitude  $\sqrt{(\frac{df(x',y')}{dx})^2 + (\frac{df(x',y')}{dy})^2} > \text{threshold}$

$$\text{and the direction } \tan^{-1} \left[ \frac{\frac{df(x',y')}{dy}}{\frac{df(x',y')}{dx}} \right] = \phi$$

where  $\phi$  is determined by the position of the point  $x', y'$  in the analytically defined shape. The maximum within the array of accumulators may be adjudged to appertain to that feature under consideration. Some techniques use a smoothed version of the array as a basis for selection.

This application requires detection of features in a feature space which is naturally contaminated by noise. Because both the central coordinates of the features need to be determined in a consistent manner and the existing noise can be quite marked the conventional Hough Transform Technique was chosen as opposed to the generalised version though directional information was included to handle the noise.

#### Definition of shapes appropriate for detection

Two parts of the eye are attractive for recognition of an eye, firstly the perimeter of the iris which is round save in cases of illness and which is attractive also because detection of a circular shape is an established task in image processing. Unfortunately should the eyes be closed or the iris be in an extreme position then detection could naturally fail. A second (distinct) part of the eye which may be employed for detection is the perimeter of the sclera, or white part, of an eye with advantage that the shape of this feature is reflected in the shape of the region below the eyebrows, thereby, in principle, allowing for eye spacing measurement even when the eyes are closed. For these reasons recognition of the shape of the perimeter of both the sclera and the iris was chosen.

An analytic shape representing the iris is naturally a circle which illustrated in fig. 1 with the expected gradient directions in each quadrant given the lighter background, An ellipse would appear the most suitable shape approximating the perimeter of the sclera. but is unsatisfactory for those parts of the eye furthest from the centre of the face. For this reason the shape of an ellipse was tailored for these parts using an exponential function. This tailored exponential shape for the right eye is illustrated in fig. 2 which again shows the gradient directions in each quadrant which are complementary to those for the iris because in this case the background is darker than the sclera.

Analytic definition of both shapes is thus:

Iris (both eyes)

$$y = y_0 \pm \sqrt{r^2 - (x - x_0)^2} \quad x_0 - r \leq x \leq x_0 + r \quad (1)$$

where  $x_0$  and  $y_0$  are the coordinates of the centre and  $r$  the radius.

Perimeter of sclera (right eye)

$$Y = y_0 \pm y_r \sqrt{1 - (x - x_0)^2 / x_r^2} \quad x_0 - r \leq x \leq x_0 \quad (2)$$

$$Y = y_0 \pm y_r \exp(-(x - x_0)^2 / r^2) \quad x_0 < x \leq x_0 + r \quad (3)$$

where  $x_r$  and  $y_r$  are the major and minor axes respectively. For the shape approximating the perimeter of the sclera of the left eye the restrictions on  $x$  in eqn. 2 and eqn. 3 are interchanged.

The general rotational transformation may be used, namely

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (4)$$

where  $x'$  and  $y'$  are the coordinates of the rotated ellipse and  $\theta$  the angle of rotation.

Calculation to determine whether each point in the image lies on a circle defined by a particular set of parameters is naturally burdensome. To speed computation it is attractive to calculate the coordinates of each point which may lie on a shape indexed by a particular set of parameters. This may take advantage of the fact that the shape of the circle is reflected around both central axes implying that the points for one quadrant only need be calculated, the points in the other quadrants being reflections around the major axes of the circle. Computation by calculating the value of the  $y$  coordinates for each increment in the  $x$  coordinate ( $x \in x_0, x_0 + 1 \dots x_0 + r$  for calculations based on the upper right hand quadrant) does not include the full set of points which may be on the circle. For this reason, computation for the value of the  $x$  coordinates for increments in the  $y$  coordinate, for each shape given  $y_n = y_0 \pm n$  ( $n \in 1, 2, \dots, y_r$ ), calculation is based on

circle  $x_n = x_0 \pm \sqrt{r^2 - n^2}$  (5)

ellipse  $x_n = x_0 \pm x_r \sqrt{1 - (n/y_r)^2}$  (6)

tailored part of ellipse  $x_n = x_0 \pm x_r \sqrt{-\log(ky_r + n)/y_r}$  (7)

where  $k = 0.368$ . A derivation of eqn. 5 is presented in the Appendix. Calculation also takes advantage of the reflection of each shape. A similar set of equations may be generated for calculation of each  $y$  coordinate given increments in the  $x$  coordinate.

Fig. 6 illustrates the match of the tailored ellipse to the perimeter of the sclera. This shape may be refined by further study to compensate for the slight mismatch of the extremity of the eye. The shape of the tailored ellipse is satisfactory also for detection of the position of the region below the eyebrows. Fortunately gradient directions at the perimeter of the eyebrow are the same as those for the perimeter of the sclera because the eyebrow is darker than the interior part of the eye. However, the centre of this shape when used to approximate the eyebrows naturally differs from the centre of the shape of the sclera.

In order to discriminate between the perimeter of the sclera and the eyebrows a criterion of a maximum count density is used for detection of the perimeter of the sclera whereas a maximum count is employed for detection of the eyebrows' shape. Hence for detection of the perimeter of the sclera the ratio

$$\text{feature}(x_0, x_r, y_0, y_r, \theta) / (x_r * y_r)$$

is maximised where  $x_0, x_r, y_0, y_r$  and  $\theta$  are as defined for eqn. 2.

#### Implementation

These techniques have been implemented using a DEC PDP LSI 11/23 microcomputer-based image processing development system which runs the SAIDIE suite of image processing software under the RT 11 operating system. This system offers facility to process images of  $128 \times 128$  pixels of 256 grey levels.

#### Application

Application of these techniques first requires generation of a gradient image in which the desired features are to be located. Fig. 4 was derived using the Sobel gradient operator applied to the image of fig. 3. The gradient magnitudes were thresholded a strength of 4 brightness values and the brightness of each point in fig. 4 depicts the direction of the gradient at that point; the brightest points depict a gradient in the direction  $0-90^\circ$ , light grey  $90-180^\circ$ , dark grey  $180-270^\circ$ , very dark grey  $270-360^\circ$  and black the background in which the gradient strength was below the threshold value. Fig. 4 emphasises the prevalence of noise contaminating the feature space and the need to



incorporate directional information in the Hough transform technique.

The procedure for location of each eye was restricted to that half of the image in which the eye should be present. Constraints on magnitude were formulated from a wide data set. Later development of such procedure and constraints awaits a generalised control strategy for the recognition procedure.

### Results

The Hough transform was applied to detect the instance of each shape in a limited data set of six subjects. A limited data set only has been used to allow fuller presentation of results. Each image of a face was obtained under the same illumination conditions. Two images of each face were taken; one with the eyes open and the other with the eyes closed. Both images were full frontal views with the centre of each face aligned approximately to the centre of the image.

Detection of the position of the iris is illustrated in fig. 5 and the result for each subject catalogued in table 1 which reveals that the mean difference between the detected position of the centre of each iris differed little from their estimated value (the mean difference in the x and y directions is zero and -0.08 pixels respectively). The measured spacing compares closely with the estimated spacing, the mean difference being 0.33 pixels.

Application of the Hough transform to detect the perimeter of the shape of the region below the eyebrows is illustrated in fig.7. The spacing measurements provided by detection the eyebrows is catalogued in table 2 which naturally indicates a wider spacing. On average this spacing appears to be 20% over that provided by measurement of spacing between the irises. A comparison of the corrected eyebrow centre - eyebrow centre spacing measurements (decreased by 20%) reveals that the mean difference between the corrected measurements and their estimated value is -1.67 pixels.

Application to determine the central coordinates of the perimeter of the sclera is illustrated in fig. 6. Spacing measurements are again catalogued in Table 2 which reveals that the spacing measurements differed on average from the estimated spacing by -1.33 pixels. It should be noted that the results for detection of the perimeter of the sclera for subject 3 and subject 5 were not derived by global application of the Hough transform. Unfortunately the shape of the perimeter of the sclera may sometimes occur in the region of the eyebrows if the eyebrows lift towards the exterior of the face. This shape can satisfy the constraints by which the perimeter of the sclera is defined for detection purposes and since it is a smaller region then the detection process can naturally be misled. For this reason, for these two subjects the application of the Hough transform was restricted to that region of the detected centre of the position of the shape of the eyebrows.

These results therefore illustrate that it is possible to derive a measurement of the spacing by detection of the position of both the iris, and the shape describing both the perimeter of the sclera and the eyebrows. It should be noted that measurement by detection of the position of the iris is the most accurate of these techniques. Detection of the perimeter of the sclera is the most sensitive of these three methods but can provide results of precision which almost equal that of iris detection. Detection of the position of the eyebrows' shape provides a measurement of eye spacing which is greater than that for other techniques but which may be used similarly to discriminate between subjects.

### Further work

Development of a control strategy for application of these techniques naturally awaits their inclusion in a full package for automated facial recognition. However, it is envisaged that application of these techniques will employ firstly detection of the position of the eyebrows with constraints derived by initial location of the perimeter of the face. This information may then be refined by recognition of either the iris (if possible) or of the shape of the perimeter of the sclera. Optimisation of the Hough transform will relieve the computational burden of application of these techniques. Recent development of the Hookes and Jeeves direct search technique has provided some encouraging results.

### Conclusions

Measurement of the eye spacing has been made using the Hough transform to locate analytically defined shapes which approximate various distinct parts of the human eye. Measurement of spacing between the centre of each iris has been made using a circle to approximate the perimeter of the iris with particular brightness profile. These measurements agreed closely with their estimated values. Measurement has also employed a shape which approximates the perimeter of both the sclera and the shape of the region below

the eyebrows. Measurement by detection of the perimeter of the sclera achieved precision approaching that of detection of each iris but with natural constraints on application. Measurement based on approximation to the eyebrows' shape was least constrained but provided measures consistently in excess of those provided by other methods. The Hough

transform technique included both gradient strength and gradient direction in order to handle noise contaminating the feature space. The Hough transform technique was also formulated in a computationally efficient manner.

Application of these techniques has led to measurements of eye spacing which may be used to discriminate between different subjects and with precision which has yet to be approached by other methods. These techniques therefore await inclusion in a full package for automated facial recognition.

#### References

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#### Appendix

##### Development of Equation 5

In order to calculate the complete set of points which lie on a circle centre  $x_0$ ,  $y_0$  with radius  $r$  either polar or cartesian rotation may be used. For cartesian coordinates.

$$\text{given } y = y_0 \pm \sqrt{r^2 - (x - x_0)^2}$$

for  $y_1 = y_0$ ,  $x_1 = x_0 - r$  (point on  $y$  axis on left side of circle).

for  $y_n = y_0 + n$  where  $n$  is an integer ( $n \in 1, 2, 3..$ )

$$n = \sqrt{r^2 - (x_n - x_0)^2}$$

where  $x_n = x_0 - r + \Delta x_n$

with solution  $\Delta x_n = r \pm \sqrt{r^2 - n^2}$

A similar method may be used to generate eqns. 6 and 7.

Table 1 Coordinates of Centre of Iris

Subject	Estimated Coordinates (±1 pixel)		Measured Coordinates	
	Left Eye	Right Eye	Left Eye	Right Eye
1	31,37	96,38	31,38	96,39
2	36,44	100,43	36,44	101,43
3	40,41	97,43	40,42	97,44
4	25,35	90,35	24,34	89,36
5	30,48	91,50	30,50	92,49
6	29,41	95,45	29,41	95,45

All measurements in pixels

Table 2 Eye Spacing Measurements

Subject	Estimated Spacing (±2 pixels)	Spacing by Iris	Spacing by Perimeter of Sclera	Spacing by Eyebrows	Eyebrow Spacing/ Estimated Spacing
1	65	65	63	80	1.23
2	64	65	63	79	1.23
3	57	57	60	72	1.26
4	65	65	60	76	1.17
5	61	62	60	76	1.25
6	66	66	64	74	1.12

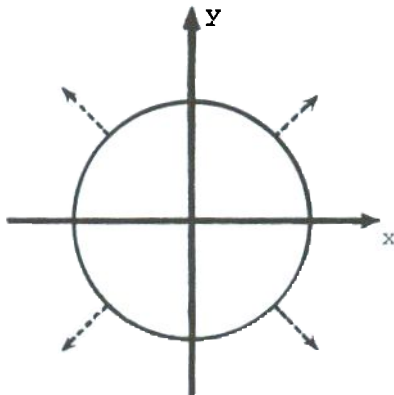


Fig. 1 Circle showing gradient directions for iris (dotted)

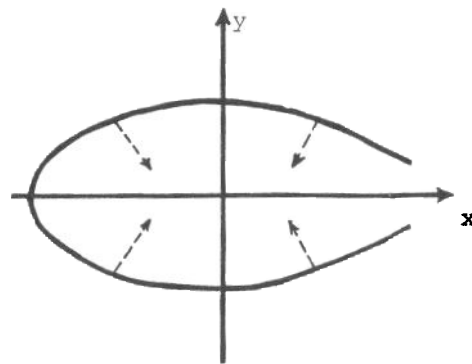


Fig. 2 Shape approximating perimeter of sclera showing gradient directions (dotted)



Fig. 3 Original image of face



Fig. 4 Gradient image of face showing gradient strength and direction



Fig. 5 Image of subject 3 showing detection of irises



Fig. 6 Image of subject 6 showing detection of perimeter of sclera



Fig. 7 Image of subject 1 showing detection of region below eyebrows